Sample Size Estimation for BMP Verification

September 11, 2014

(Shortened from August 14, 2014)

TETRA TECH

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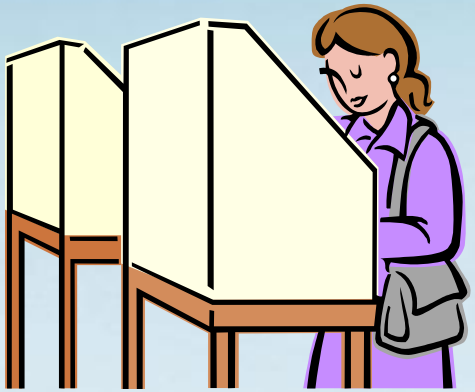
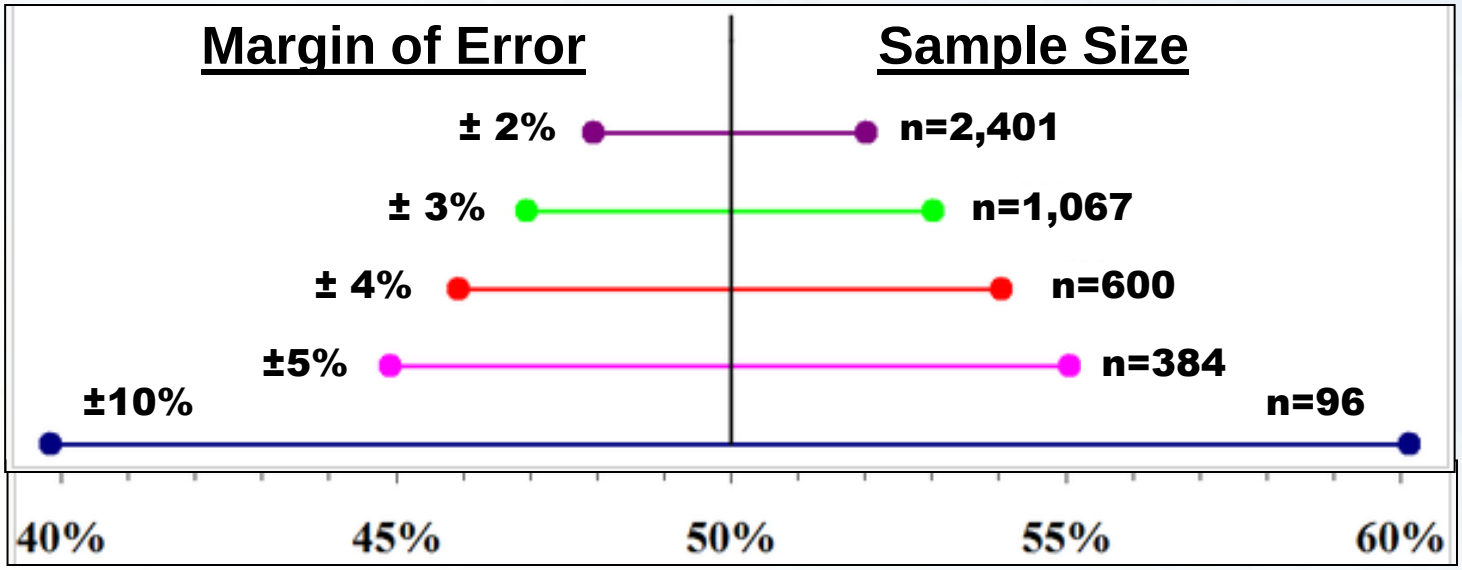
Sample Size Estimation

- Objective/Management Goal
  - Document the percentage of BMPs that are still in place and functioning properly
  - Inform Bay model simulation updates
- Target Populations
  - BMP implemented through state cost-share program
  - BMP implemented through CAFO permits
  - BMP implemented voluntarily without cost-share
  - Resource improvement practice

Binomial Distribution



- Binomial Distribution
  - Are the BMPs still there?
    - Yes/No
  - Are the BMPs still functioning properly?
    - Yes/No
- Sample Size—just like political polls



Source: http://en.wikipedia.org/wiki/Margin_of_error#Calculations_assuming_random_sampling

Binomial Distribution

Standard Sample Size Equation

$$n_o = \frac{\left(Z_{1-\alpha/2}\right)^2 pq}{d^2}$$

Political Poll Example

$$96 = \frac{(1.96)^2(0.5)(0.5)}{(0.10)^2}$$

Finite Population Correction

$$n = \frac{n_o}{(1+\varphi)}$$

N = total number of population units in sample population

n_o = preliminary estimate of sample size (sample size for large N)

$Z_{1-\alpha/2}$ = value corresponding to cumulative area of 1-α/2 using the normal distribution

p = proportion of "yes" responses

q = proportion of "no" responses (i.e., 1-p)

d = allowable error (margin of error)

φ = n_0/N unless otherwise stated

n = number of samples (adjusted for finite population)

Observations

- Improved precision
  - More sampling
- Reduce sampling costs
  - Lower confidence level (e.g., 95% CI → 90% CI)
  - Increased allowable error, d, (e.g., ±10% → ±15%)
- Less sampling is needed to maintain precision if the percentage of BMPs maintained is closer to 100%
  - A priori knowledge is important
  - 50% BMP maintenance is a conservative assumption
  - But don’t overestimate
- Finite Populations
  - Sampling from small populations can result in large errors.

Potential Application

- Precision Statement
  - Estimate the percentage of BMPs maintained, **p**, to within **±d%** using a **X%** confidence level.
- Example:
  - The percentage of BMPs maintained is 85% ±10% with a 95% confidence level, or
  - The range of maintained BMPs is 75-95% with a 95% confidence interval.
- Worked Example
  - **p**: No information (50%), Good (70%), Excellent (85%)
  - **±d**: 5%, 10%, and 15%
  - **X%**: 90% and 95%

Work Example—Sample Size (n)

95% Confidence Level

p		±d	Large N	100	200	600	1000	1,500	2,000
No Information	50%	5%	385	80	132	235	278	307	323
	50%	10%	97	50	66	84	89	92	93
	50%	15%	43	31	36	41	42	42	43
Good Maintenance	70%	5%	323	77	124	210	245	266	279
	70%	10%	81	45	58	72	75	77	78
	70%	15%	36	27	31	34	35	36	36
Excellent	85%	5%	196	67	99	148	164	174	179
	85%	10%	49	33	40	46	47	48	48
	85%	15%	22	19	20	22	22	22	22

90% Confidence Level

p		±d	Large N	100	200	600	1000	1,500	2,000
No Information	50%	5%	271	74	116	187	214	230	239
	50%	10%	68	41	51	62	64	66	66
	50%	15%	31	24	27	30	31	31	31
Good Maintenance	70%	5%	228	70	107	166	186	198	205
	70%	10%	57	37	45	53	54	55	56
	70%	15%	26	21	24	25	26	26	26
Excellent	85%	5%	138	58	82	113	122	127	130
	85%	10%	35	26	30	34	34	35	35
	85%	15%	16	14	15	16	16	16	16

Work Example—Sample Level (n/N)

>20%	●
10-20%	●
5-10%	●
<5%	●

95% Confidence Level

p		±d	Large N	100	200	600	1000	1,500	2,000
No Information	50%	5%	385	80%	66%	39%	28%	20%	16%
	50%	10%	97	50%	33%	14%	9%	6%	5%
	50%	15%	43	31%	18%	7%	4%	3%	2%
Good Maintenance	70%	5%	323	77%	62%	35%	25%	18%	14%
	70%	10%	81	45%	29%	12%	8%	5%	4%
	70%	15%	36	27%	16%	6%	4%	2%	2%
Excellent	85%	5%	196	67%	50%	25%	16%	12%	9%
	85%	10%	49	33%	20%	8%	5%	3%	2%
	85%	15%	22	19%	10%	4%	2%	1%	1%

90% Confidence Level

p		±d	Large N	100	200	600	1000	1,500	2,000
No Information	50%	5%	271	74%	58%	31%	21%	15%	12%
	50%	10%	68	41%	26%	10%	6%	4%	3%
	50%	15%	31	24%	14%	5%	3%	2%	2%
Good Maintenance	70%	5%	228	70%	54%	28%	19%	13%	10%
	70%	10%	57	37%	23%	9%	5%	4%	3%
	70%	15%	26	21%	12%	4%	3%	2%	1%
Excellent	85%	5%	138	58%	41%	19%	12%	8%	7%
	85%	10%	35	26%	15%	6%	3%	2%	2%
	85%	15%	16	14%	8%	3%	2%	1%	1%

What is the basis for using the initial estimate of 50% as the standard for presence or functioning of BMPs?

A: 50% isn't a standard per se. Rather, the assumption of 50% is the most conservative estimate in terms of sample size calculation. If you have no better information, it's the safest approach to meeting your precision requirements.

What is the basis for recommending use of a percentage from previous studies?

A: If a jurisdiction has information from past assessments to indicate that the actual percentage of BMPs present or functioning is greater than or less than 50%, then a smaller sample size can be used to achieve the same precision requirements.

Because the formula does not establish a defined allowable error (d) or confidence level (α), then the Ag Work Group should do so. Does that not return us to the debate over the prior 80% confidence level that was rejected?

A: Assumptions of allowable error or confidence level will exist regardless of the approach taken, whether stated explicitly or implicit in the decision making. For example, assuming that either a 5% or 10% sampling is adequate presumes an allowable error and confidence level even if they have not been stated. Our approach starts with those decisions and then does the math based on those choices.

$$\% = 100 \times \frac{n}{N}$$

$$n = \frac{n_o}{(1+\varphi)}$$

$\varphi = n_0/N$

$$n_o = \frac{\left(Z_{1-\alpha/2}\right)\ pq}{d^2}$$

It is unclear why the total number of BMPs initially reported as being present and operating is not the baseline for deriving the sample numbers. Am I simply confused?

A: The total number of BMPs initially reported as being implemented and operational is N. The sampling approach calculates the number of those BMPs that would need to be sampled to confirm continued presence and operation given a specified allowable error and confidence level.

N = total number of population units in the sample population

= the total number of BMPs initially reported as being implemented and operational

The procedure incorporates an "estimate of the total population (N) from which the sample is taken" and argues that this estimate "can be based on records of BMP implementation." How? Isn't verification of implementation what the process is trying to determine?

A: The total number of BMPs that you wish to check via follow-up inspection = N and the jurisdiction would perform a follow-up inspection at n sites. The approach assumes knowledge of where and how many BMPs were originally implemented. If there is no advance knowledge of where or how many BMPs were implemented, it is unclear how a state would ensure that an unbiased sample of 5% or 10% could be implemented. If the state cannot determine N, then it would be conservative to assume a large N and not take advantage of the finite population correction term that can reduce sample size. The state would then need to develop an approach for randomly selecting sites that doesn't rely on an inventory of BMPs.

There needs to be a clear linkage between the formula and the verification standards we are establishing. For example, who does the sampling and how is it done when using the formula?

A: The August 14, 2014, draft of *Statistical Sampling Approach for Initial and Follow-Up BMP Verification* attempts to address this linkage by inclusion of a section on practical sampling considerations. It also includes a discussion on how to apply the approach to the Chesapeake Bay Program and provides a generalized example for direct use by the jurisdictions. Selection of appropriate verification methods at sites selected using this approach is addressed in the agricultural BMP verification guidance.

I am assuming that if a state is to determine how to use this formula, it would be reviewed by the BMP Review Panel and EPA. We would, as we discussed, need to insure this linkage, also, in any verification guidance?

A: This is not a question for Tetra Tech, but we can provide assistance in applying the formula. To that end, the examples provided here and in the August 14, 2014, draft of *Statistical Sampling Approach for Initial and Follow-Up BMP Verification* provide jurisdictions with "look-up" tables to determine sampling sizes.

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