Modeling unsteady lumped transport with time-varying transit time distributions

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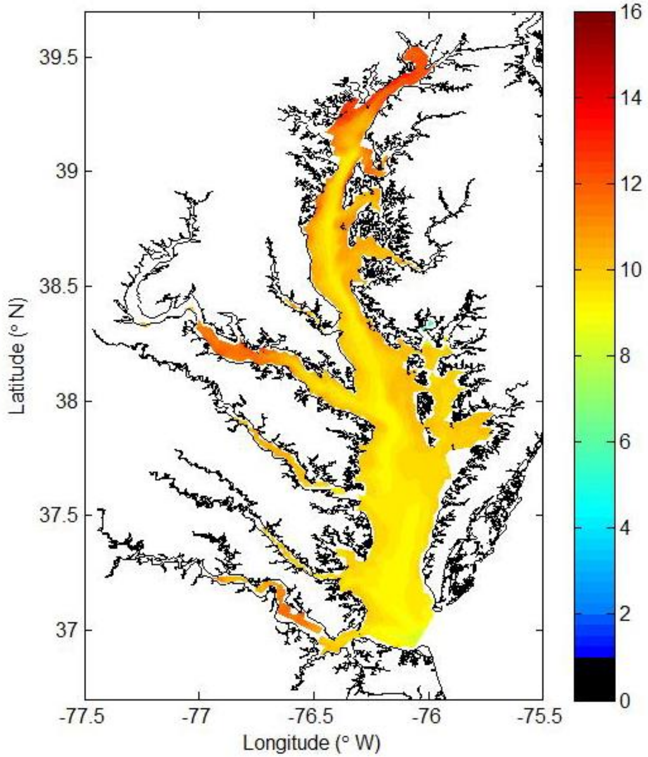
Two views of transport

Eulerian

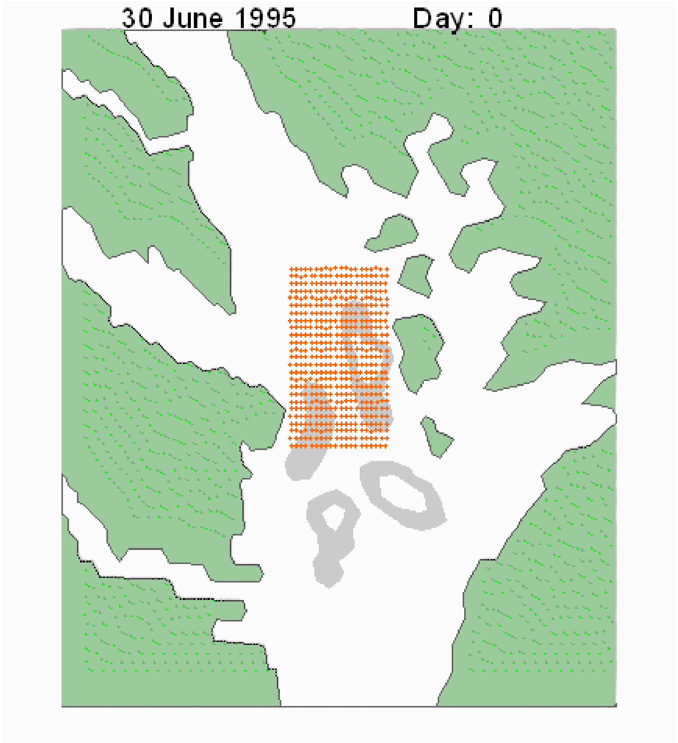
Concentration at fixed points



DO above the bed
Courtesy of Jeremy Testa, UMCES

Lagrangian

Parcels moving through space

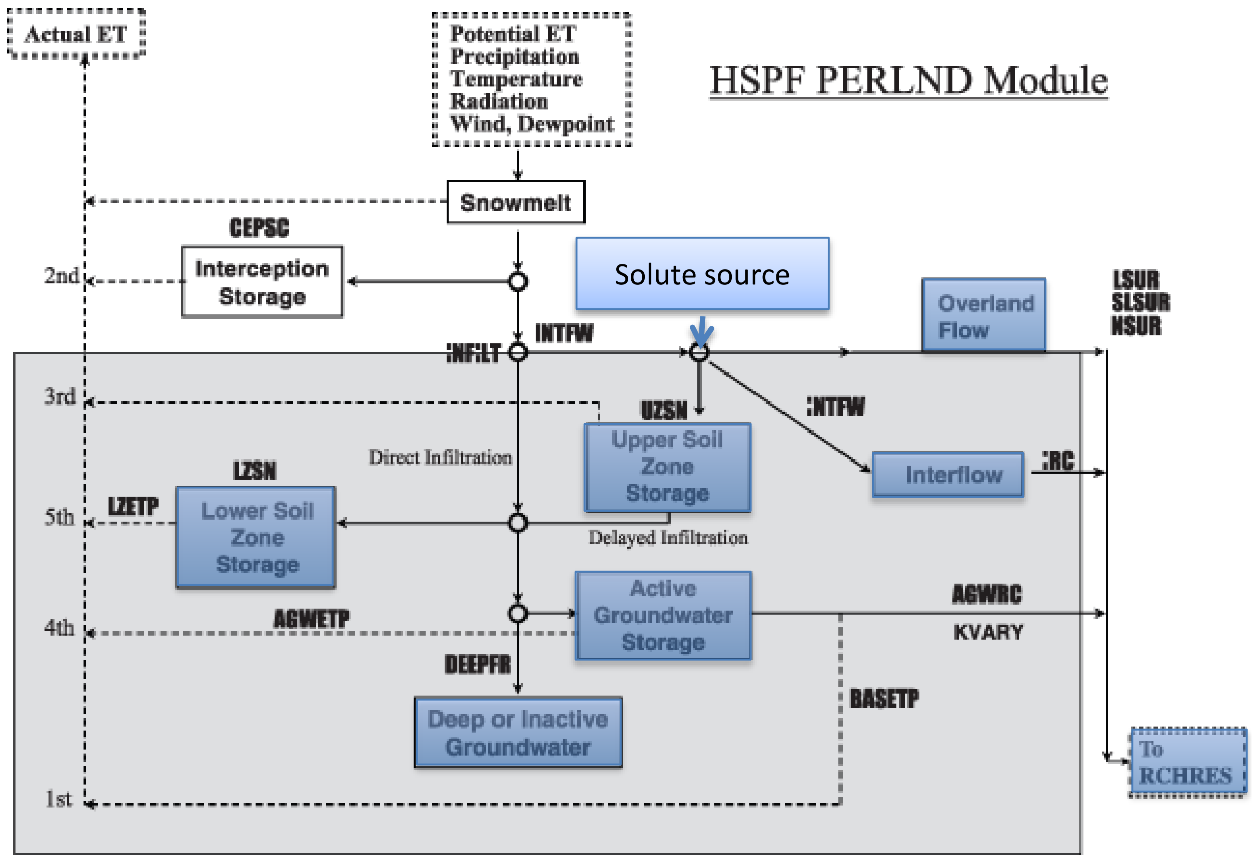


LTRANS Larval transport model (North et al 2006)
from www.usglobec.org

Spatially aggregated versions

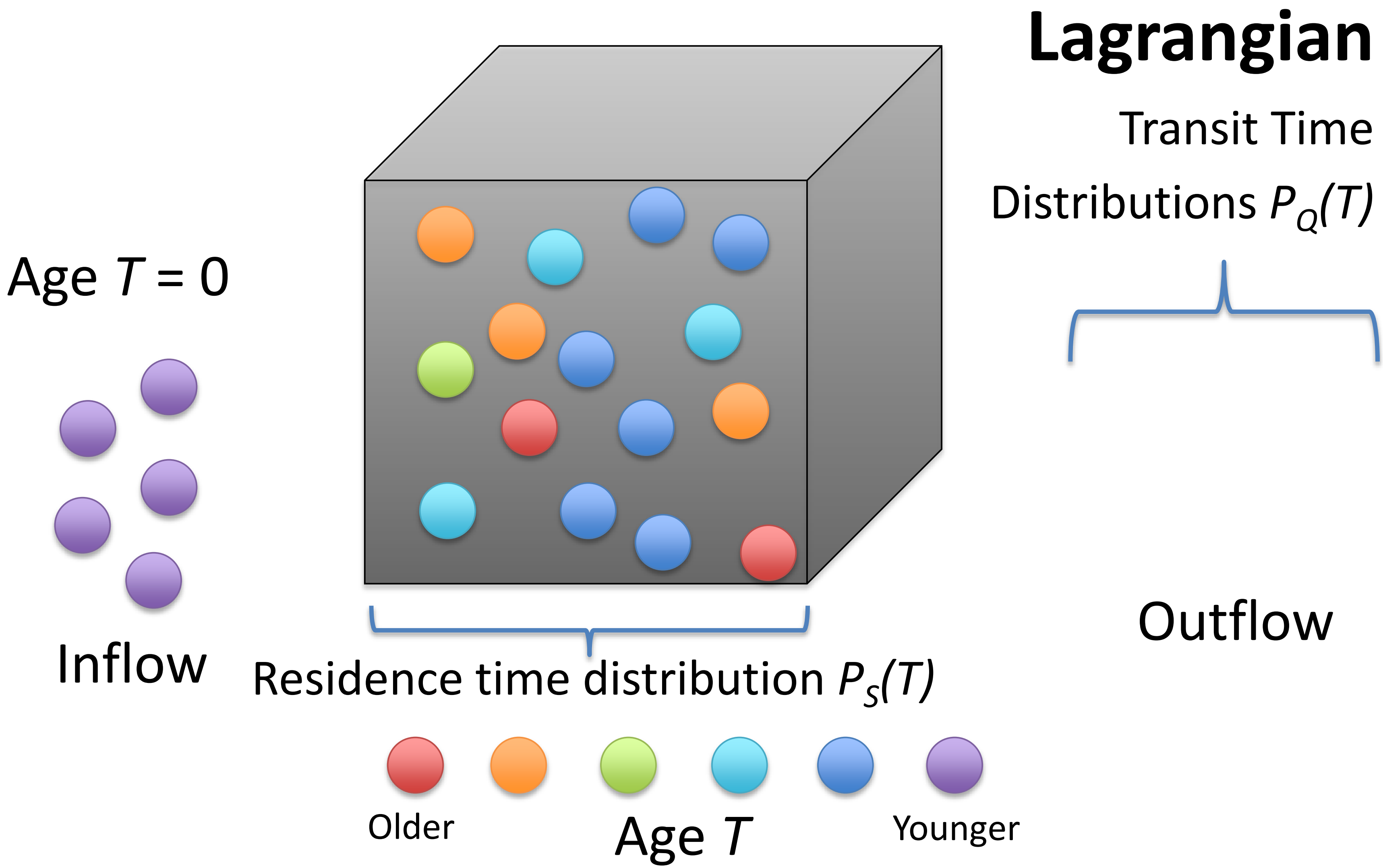
Eulerian

Mixing box models (built on lumped flow model)



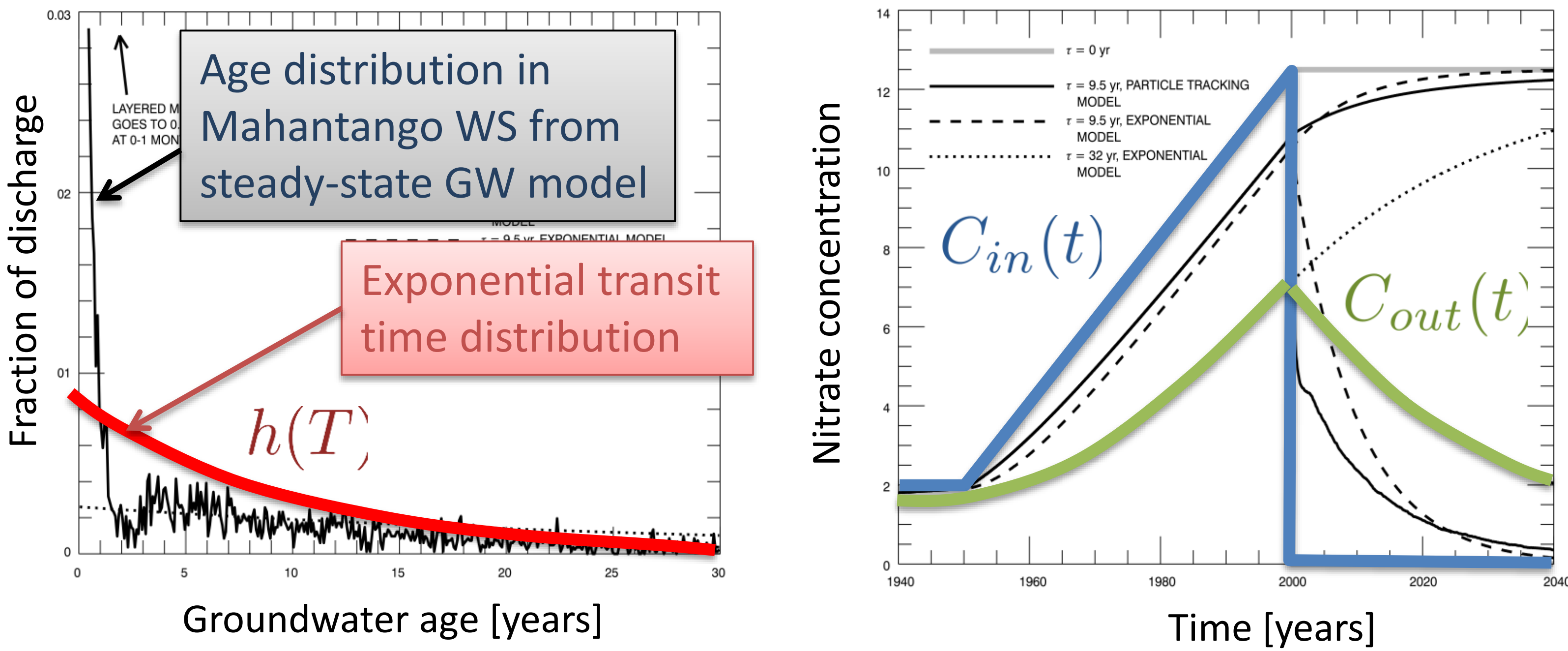
USGS 2005

Spatially aggregated versions



If Q is steady-state

Lindsey et al (2003)



"Convolution"

$$C_{out}(t) = \int_{-\infty}^{t} C_{in}(\tau) h(t-\tau) d\tau$$

Advantages of TTD approach

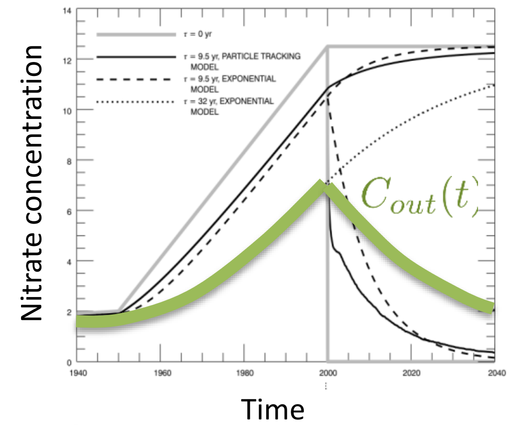
- Few parameters

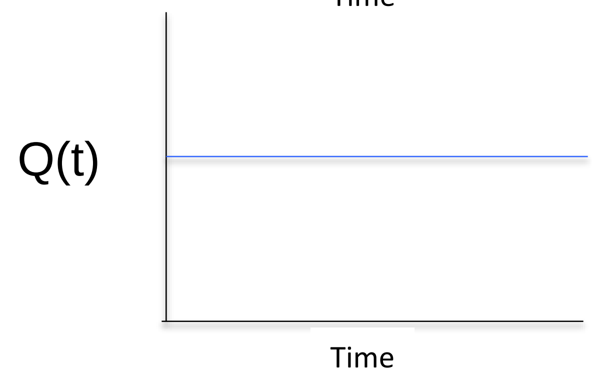
- Captures ‘emergent’ effect of heterogeneity

- Can reproduce anomalous phenomena

- ‘Steady-state’ requirement restricts modeling applications

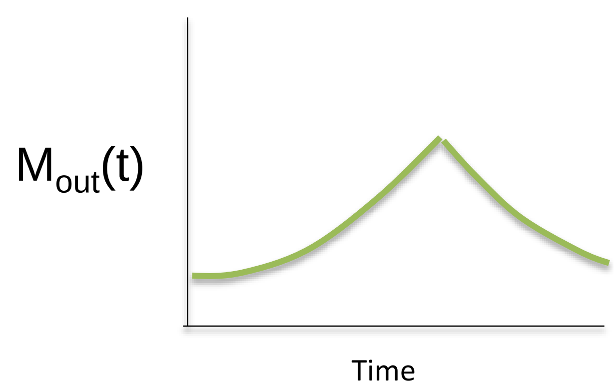
- Non-steady flow *will not conserve mass*





Steady discharge

$$M_{out}(t) = Q(t) \times C_{out}(t)$$



- ‘Steady-state’ requirement restricts modeling applications

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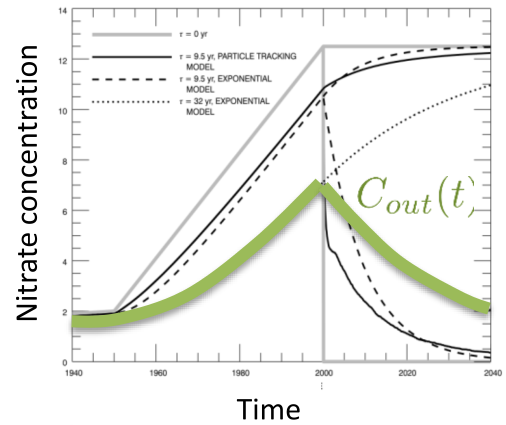
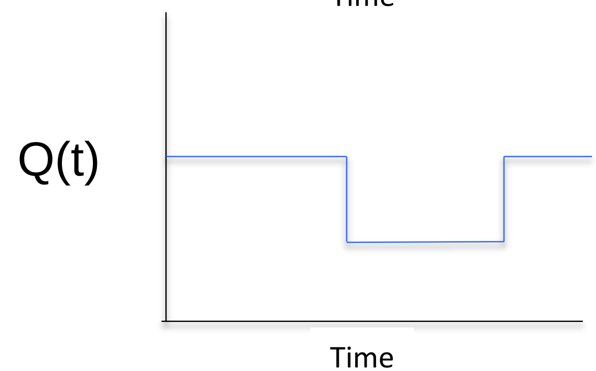
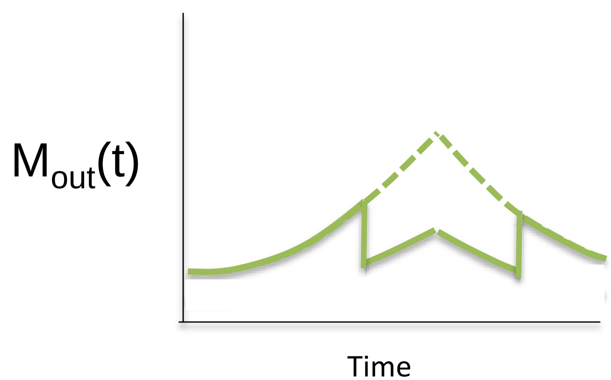




Un-steady discharge

$$M_{out}(t) = Q(t) \times C_{out}(t)$$



Missing mass!

Sources of variability in TTD

Climate change

1. Temporal variability of input (Precip & irrigation)
2. Water balance partitioning variability (*ET* vs *Q*)
3. Hydrologic pathway variability
   (e.g. overland flow under wet conditions)
4. Long term pathway change
   (e.g. urbanization)

Land-use change

International Journal of Applied Radiation and Isotopes. 1977. Vol. 28. pp. 855–860. Pergamon Press. Printed in Great Britain

Residence Time Distributions of Variable Flow Processes

ANTTI J. NIEMI

Helsinki University of Technology, Control Engineering Laboratory, Otaniemi, Finland

(*Received* 19 *May* 1977)

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WATER RESOURCES RESEARCH, VOL. 46, W03514, doi:10.1029/2009WR008371, 2010

Transport in the hydrologic response: Travel time distributions, soil moisture dynamics, and the old water paradox

Gianluca Botter,[1] Enrico Bertuzzo,[2] and Andrea Rinaldo[1,2]

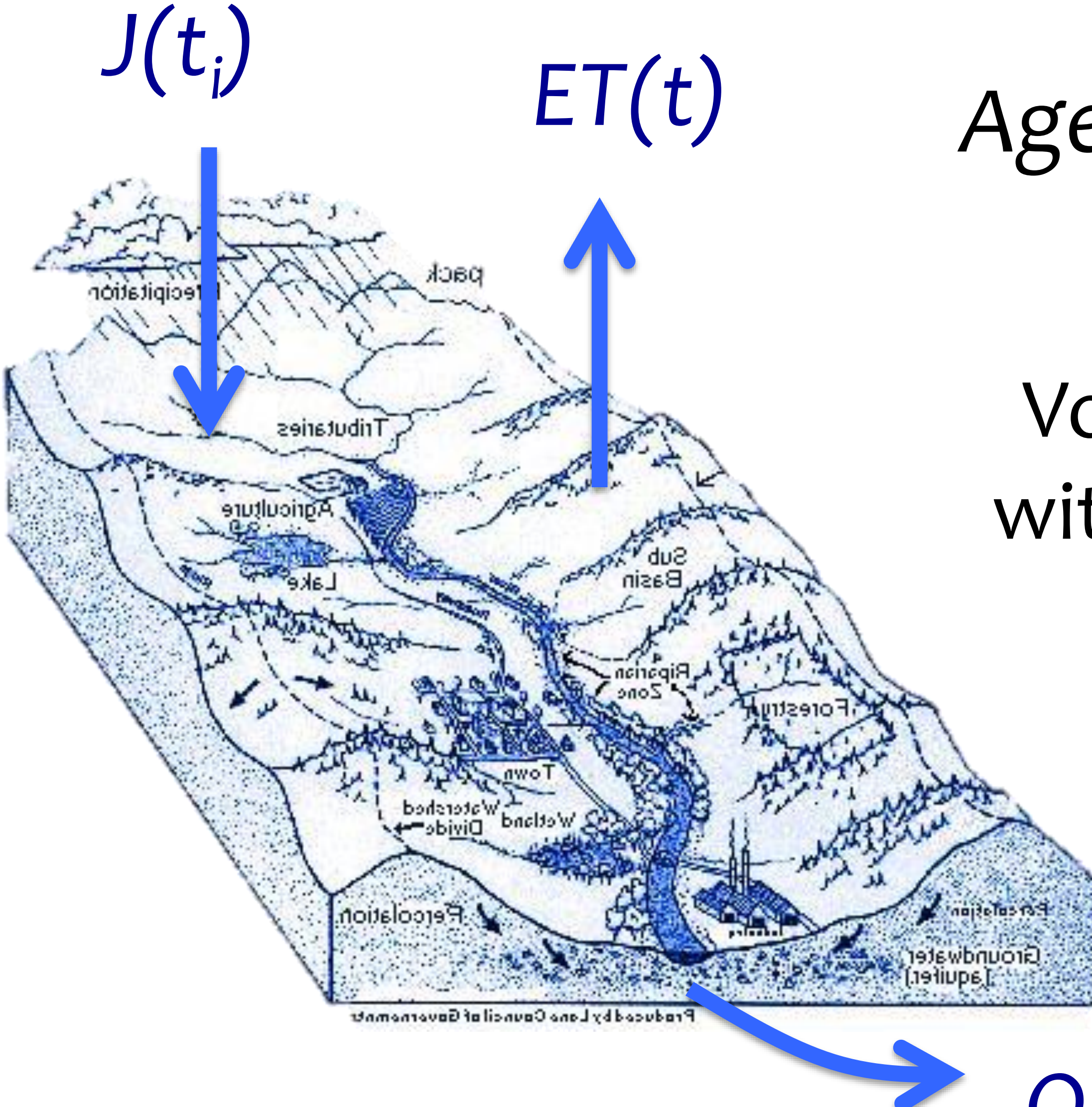
Received 8 July 2009; revised 23 October 2009; accepted 29 October 2009; published 12 March 2010.

[1] We propose a mathematical framework for the general definition and computation of travel time distributions defined by the closure of a catchment control volume, where the input flux is an arbitrary rainfall pattern and the output fluxes are green and blue water flows (namely, evapotranspiration and the hydrologic response embedding runoff production through soil water dynamics). The relevance of the problem is both practical, owing to implications in hydrologic watershed modeling, and conceptual for the linkages and the explanations the theory provides, chiefly concerning the role of geomorphology, climate, soils, and vegetation through soil water dynamics and the treatment of the so-called old water paradox. The work focuses in particular on the origins of the conditional and time-variant nature of travel time distributions and on the differences between unit hydrographs and travel time distributions. Both carrier flow and solute matter transport in the control volume are accounted for coherently. The key effect of mixing processes occurring within runoff production is also investigated, in particular by a model that assumes that mobilization of soil water involves randomly sampled particles from the available storage. Travel time distributions are analytically expressed in terms of the major

There is now a rigorous and convenient theoretical framework for time-variable TTD

SAS - StorAge Selection functions

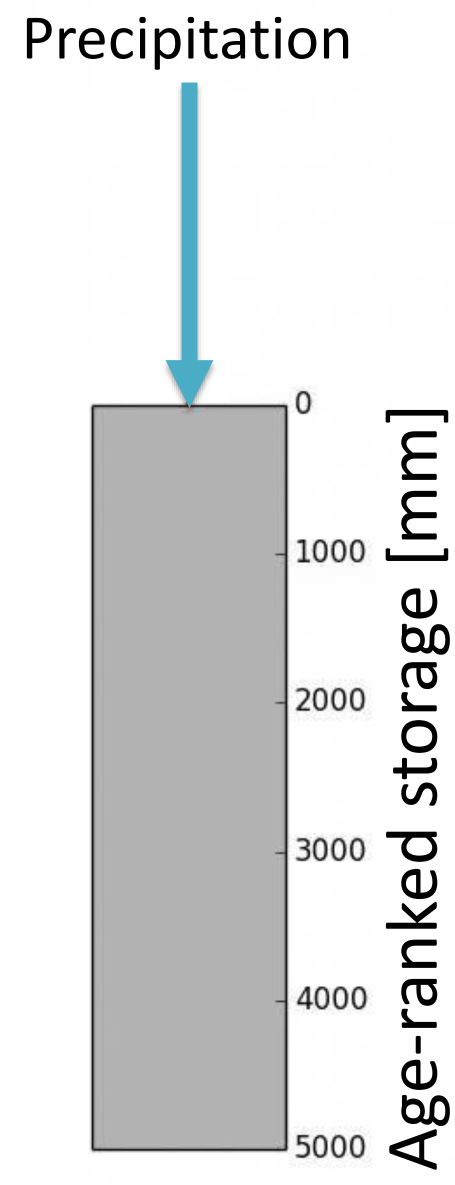
- aSAS - Botter *et al*, (2011)
- fSAS - Van der Velde *et al,* (2012)
- rSAS - Harman (in review)



Age-ranked storage

$$S_T(T,t)$$

Volume in storage with age less than *T* at time *t*


Precipitation
0
1000
2000
3000
4000
5000
Age-ranked storage [mm]

rank StorAge Selection (rSAS)

Conservation law for S_T

$$\frac{\partial S_T}{\partial t} = J(t) - Q(t)\overleftarrow{P}_Q(T|t) - ET(t)\overleftarrow{P}_{ET}(T|t) - \frac{\partial S_T}{\partial T}$$

Hydrologic timeseries

Closure relations

Discharge TTD

ET TTD

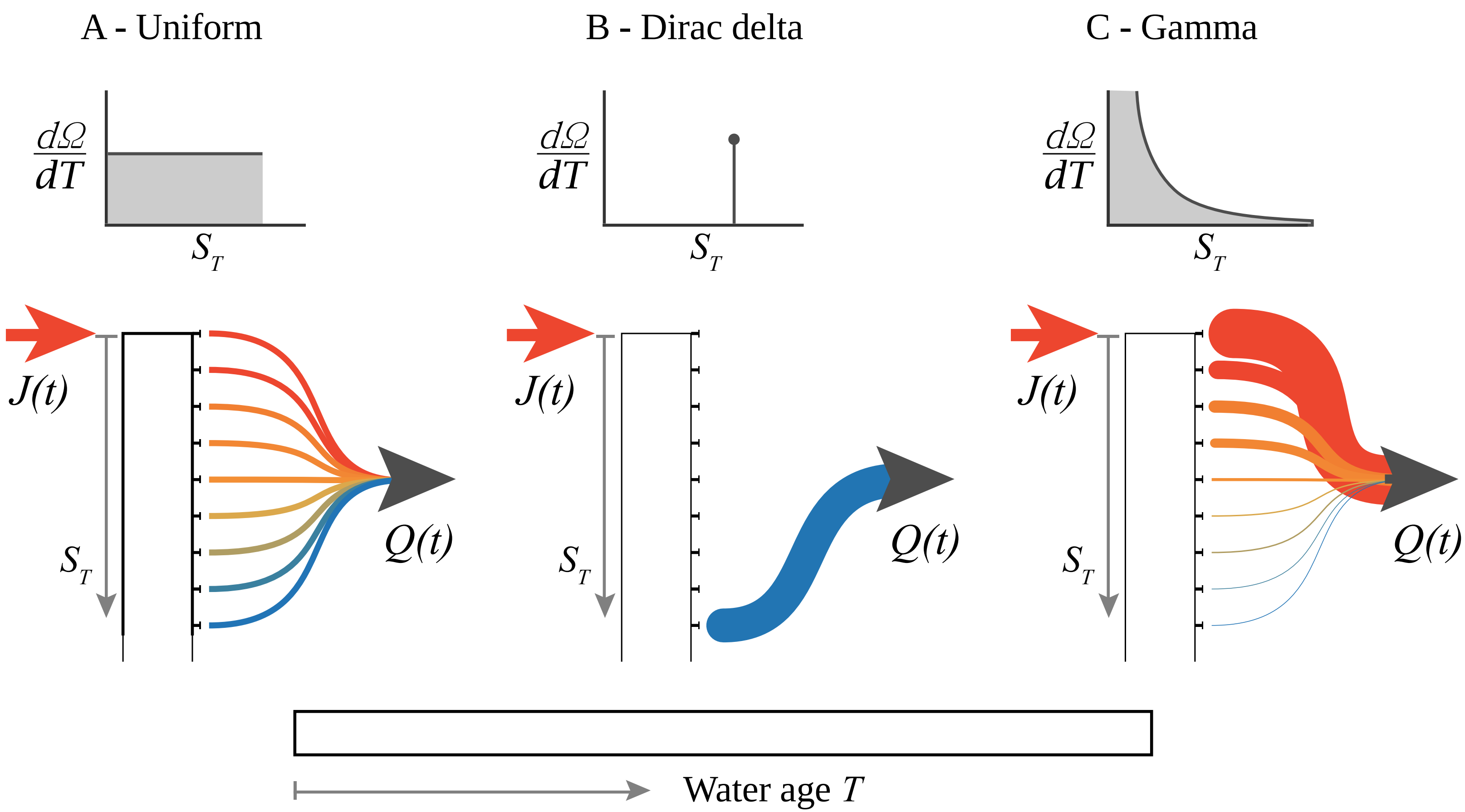
$$\overleftarrow{p}_Q(T,t) = \frac{\partial S_T}{\partial T}\omega_Q(S_T,t)$$

$$\overleftarrow{p}_{ET}(T,t) = \frac{\partial S_T}{\partial T}\omega_{ET}(S_T,t)$$

Discharge rSAS function

ET rSAS function

Shape of the rSAS function determines transport

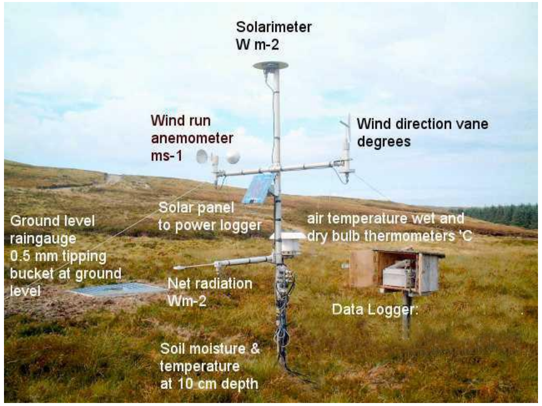


Application to Lower Hafren stream

Long-term precip + stream chloride

~27 years weekly

~3 years daily samples



Thanks to Jim Kirchner for gap-filled hydroclimatic data

Lower Hafren stream

~3.67 km^2

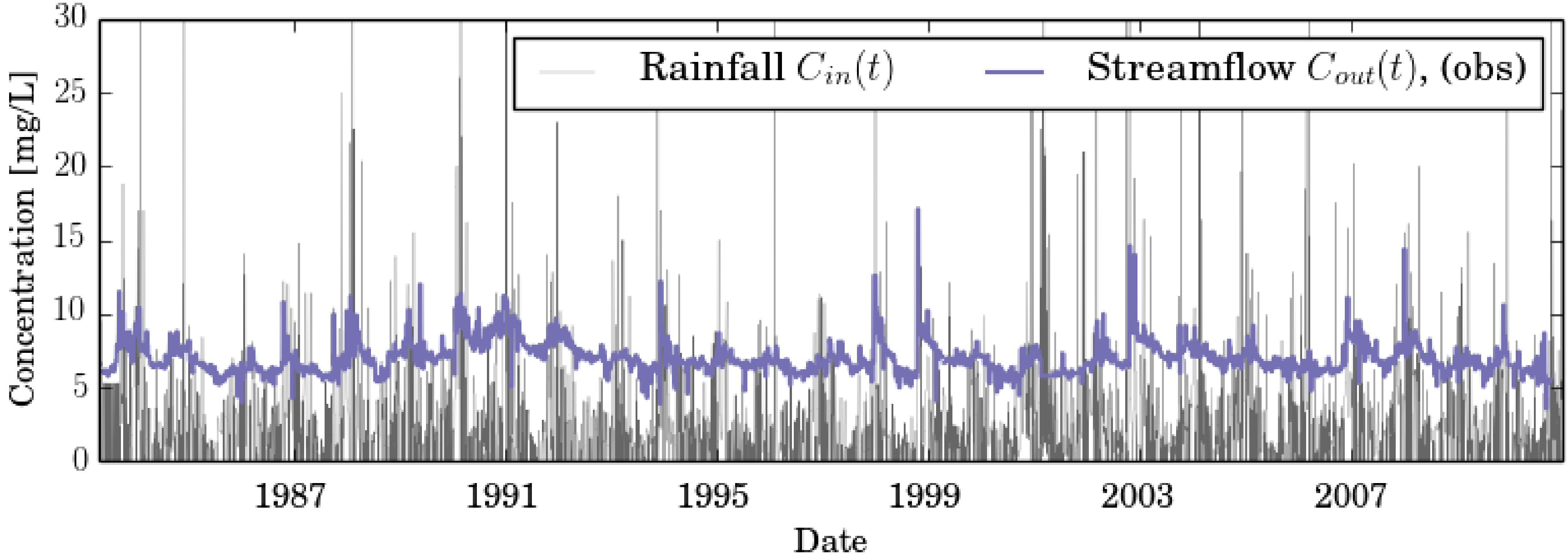
48% forest (mainly lower part)

68% peatland (mostly uplands)

Faulted mudstones and slate

Photos by Tomáš Formánek

Extraordinary public dataset of stream and precip chemistry:
Long-term (20+ years of weekly samples)
and high frequency (3 years of daily, 2 years of 7-hourly)



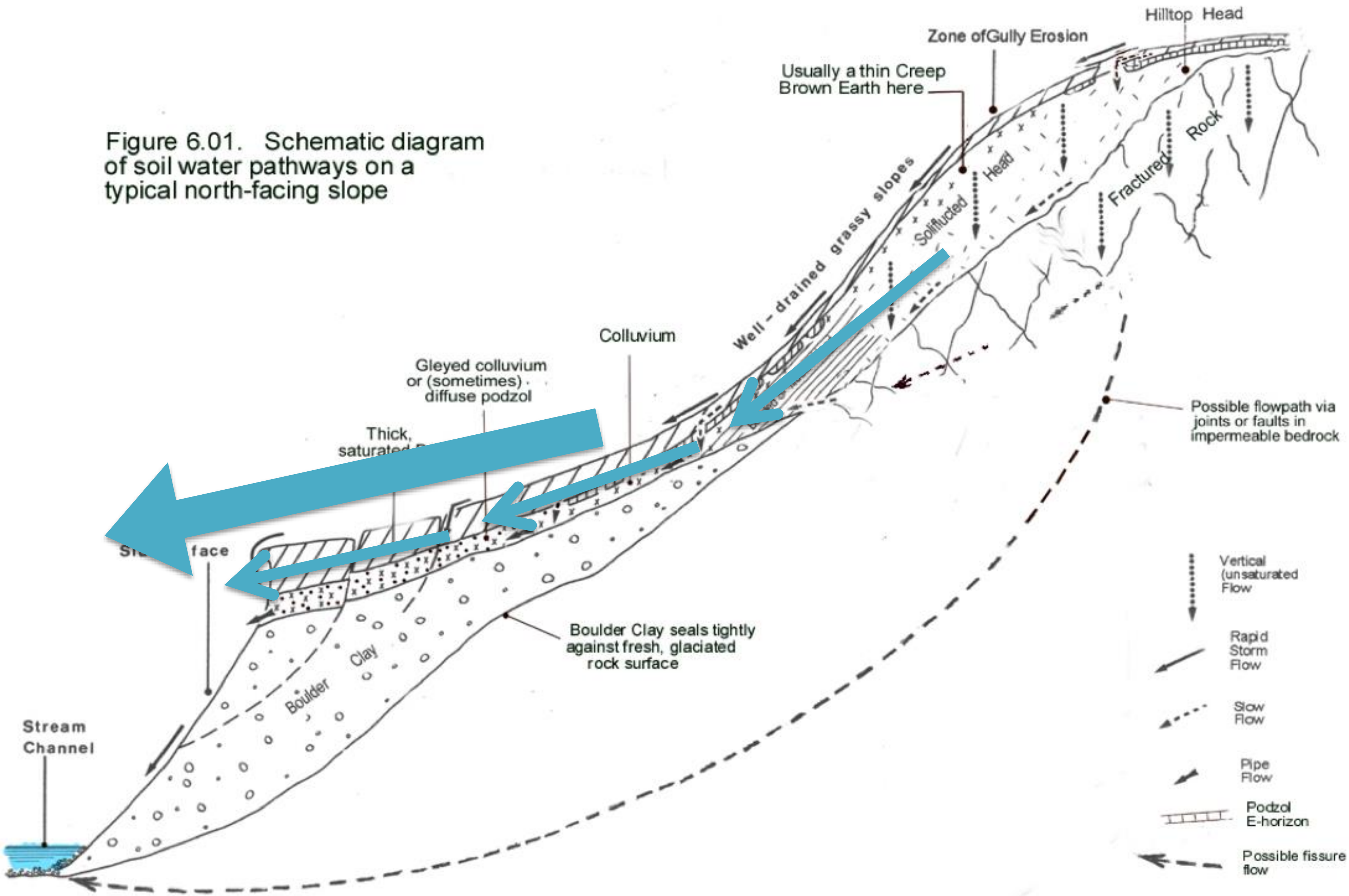


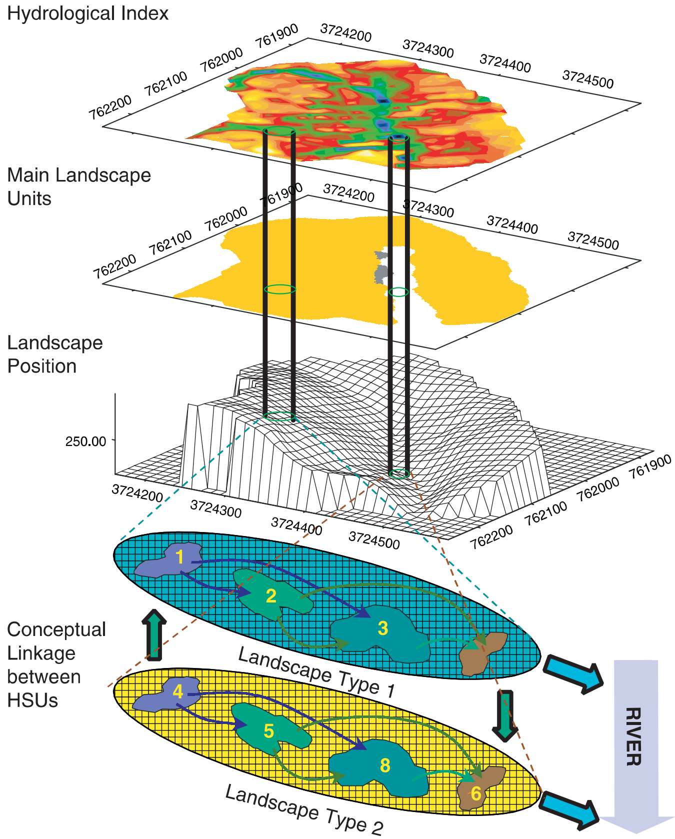
Figure 6.01. Schematic diagram of soil water pathways on a typical north-facing slope
Hilltop Head
Zone of Gully Erosion
Usually a thin Creep Brown Earth here
Head
Solifluction
Fractured Rock
Well - drained grassy slopes
Colluvium
Gleyed colluvium or (sometimes) diffuse podzol
Thick, saturated
face
Possible flowpath via joints or faults in impermeable bedrock
Boulder Clay
Boulder Clay seals tightly against fresh, glaciated rock surface
Stream Channel
Vertical (unsaturated Flow
Rapid Storm Flow
Slow Flow
Pipe Flow
Podzol E-horizon
Possible fissure flow

Bell (2005)

Previous modeling by Page et al (2007)

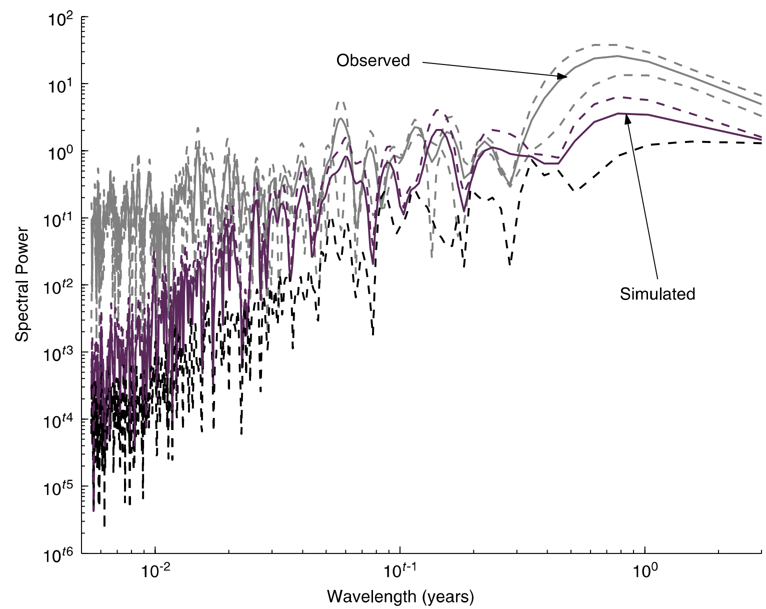
Dynamic TOPMODEL

18+ calibrated parameters



"Behavioral" NSE > **0.15**

703 of 60,000 parameter sets

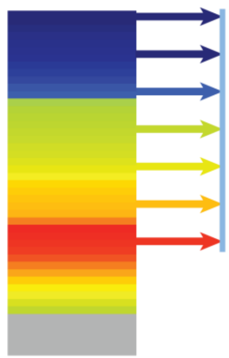


Could not capture spectral structure

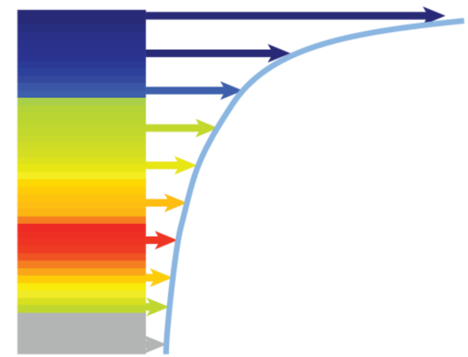
Application of the Omega function

- Watershed modeled as a single control volume
- No hydrologic model
- Assumed functional forms for Ω_Q & Ω_{ET} PDFs
- Parameters fit by minimizing RMSE of predicted stream $[Cl^-]$

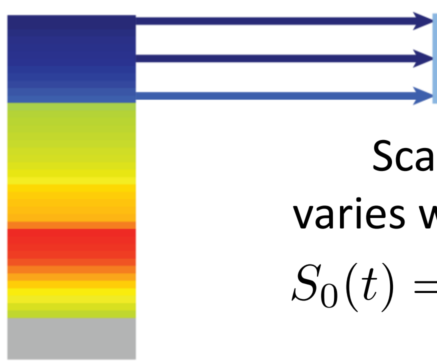
rSAS function parameterization



Fixed uniform



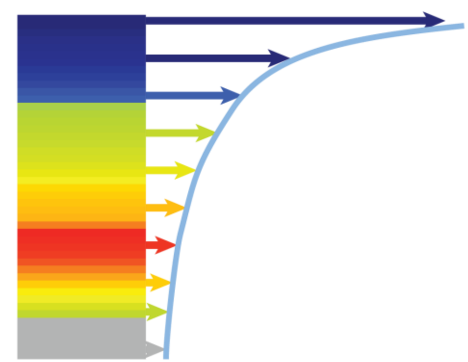
Fixed gamma



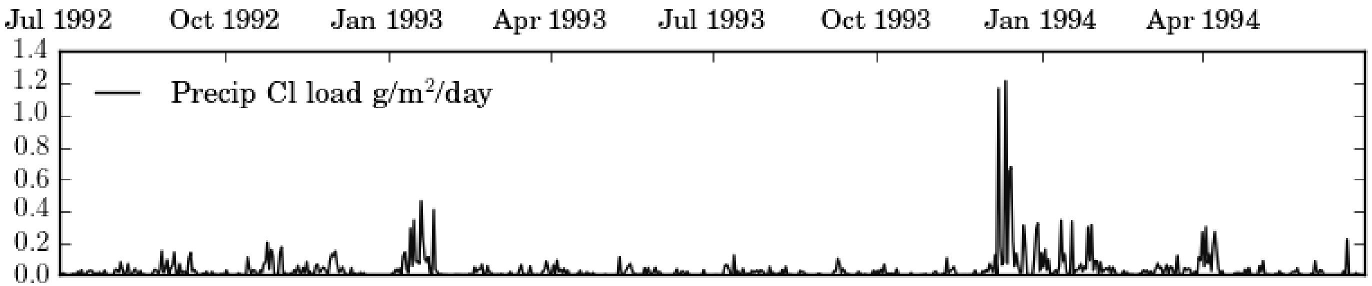
Storage-dependent uniform

Scale parameter S_0
varies with relative storage

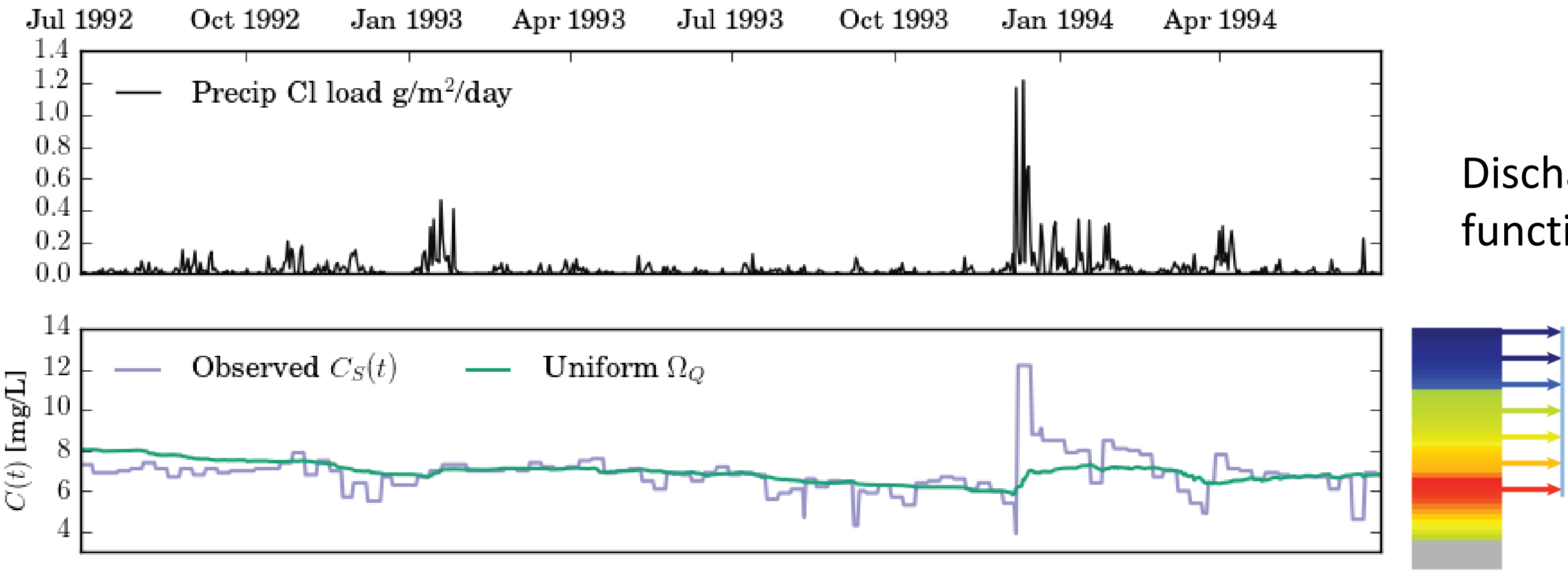
$$S_0(t) = \lambda\left(\Delta S(t) - \Delta S_c\right)$$



Storage-dependent gamma



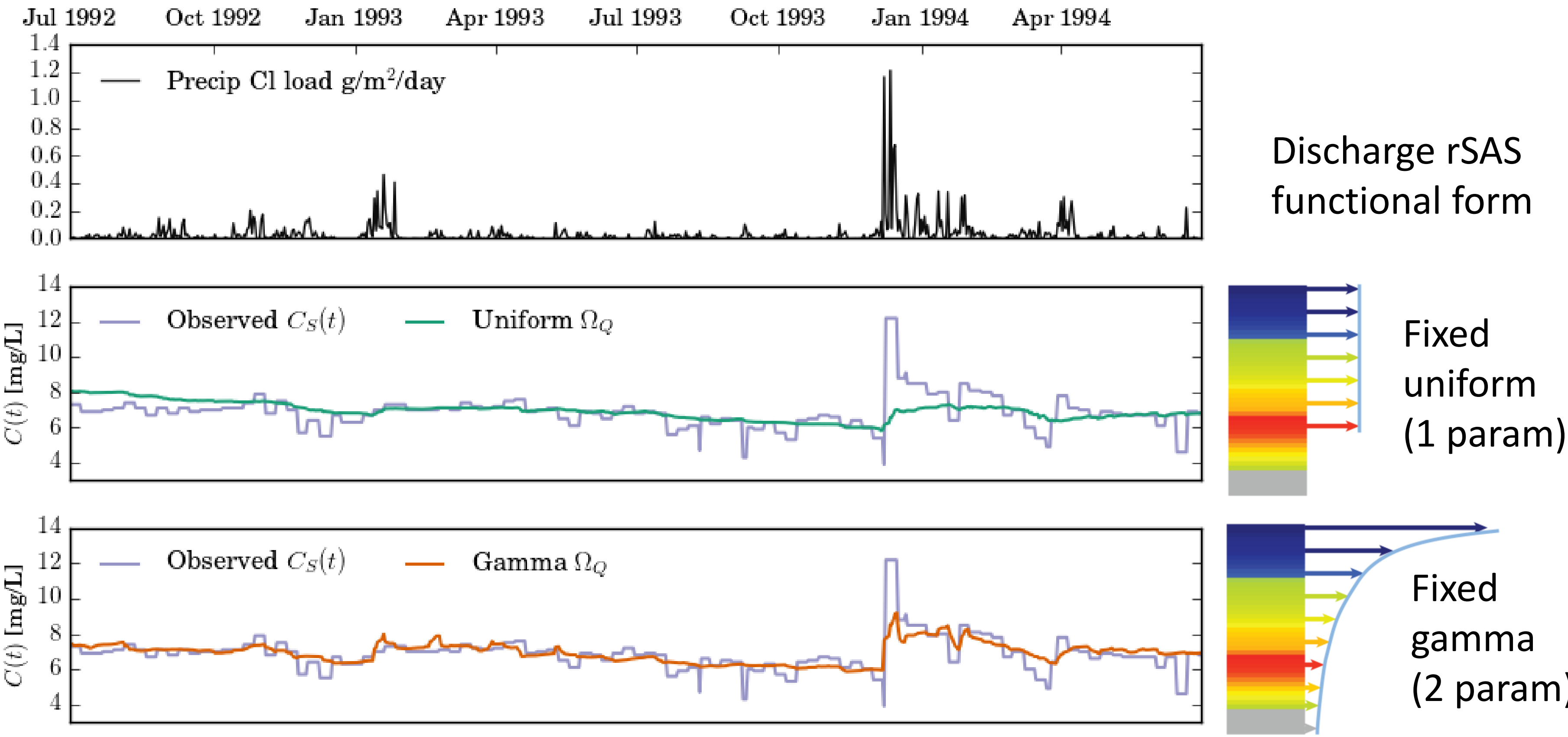
Jul 1992
Oct 1992
Jan 1993
Apr 1993
Jul 1993
Oct 1993
Jan 1994
Apr 1994
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Precip Cl load g/m2/day



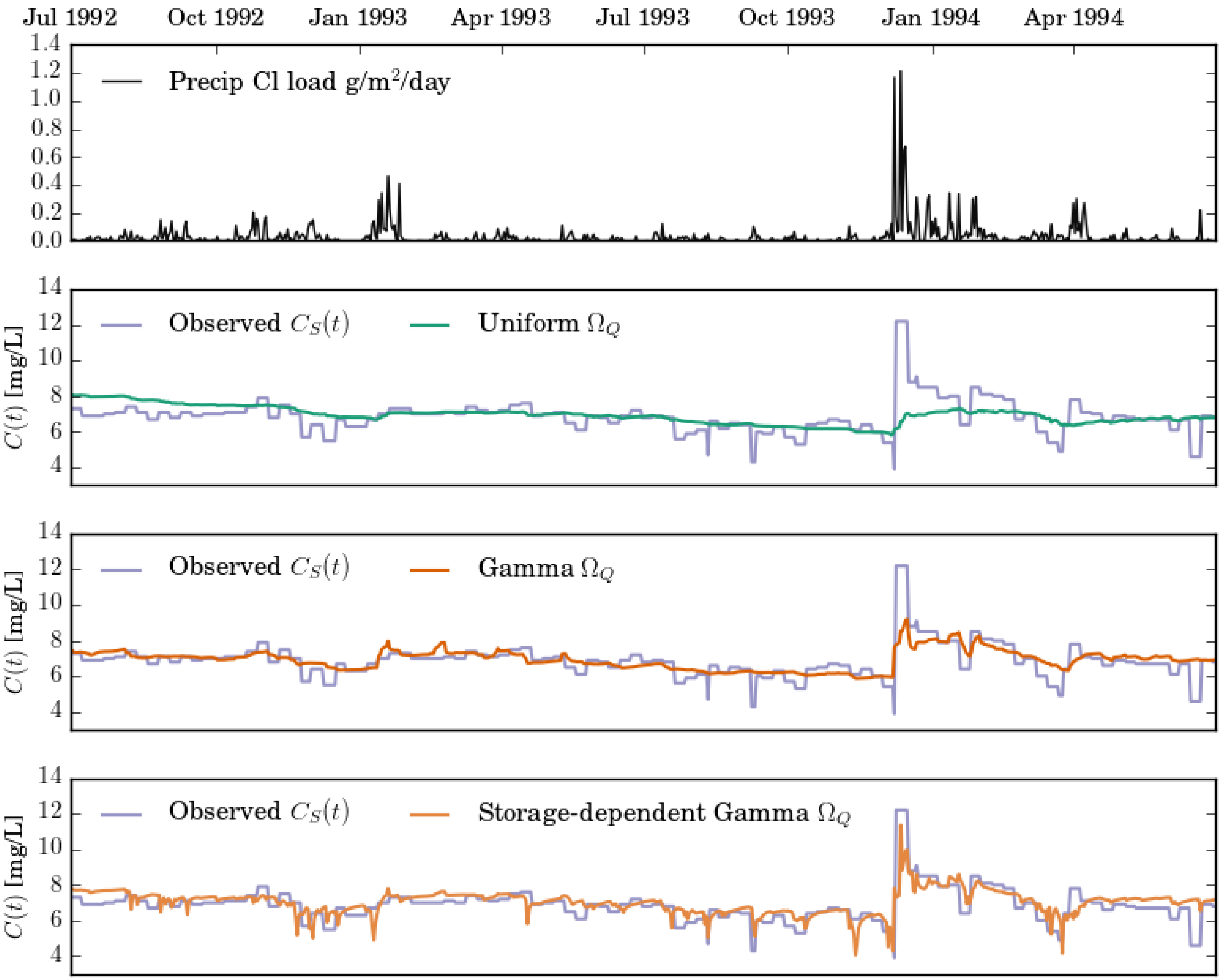
Discharge rSAS functional form

Fixed uniform (1 param)

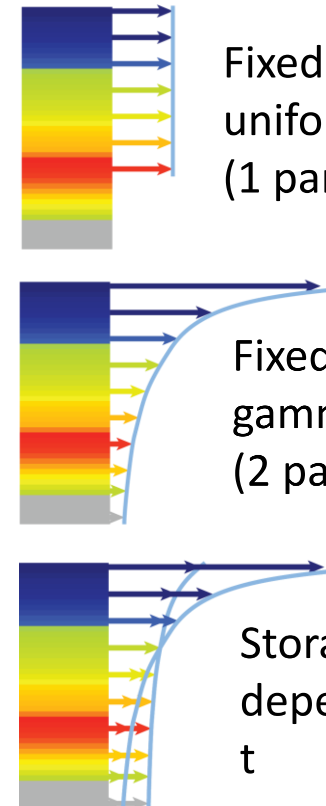
Nash-Sutcliffe Efficiency = **0.35** calibration **0.13** validation



Nash-Sutcliffe Efficiency = **0.54** calibration **0.48** validation



Discharge rSAS functional form



Fixed uniform (1 param)

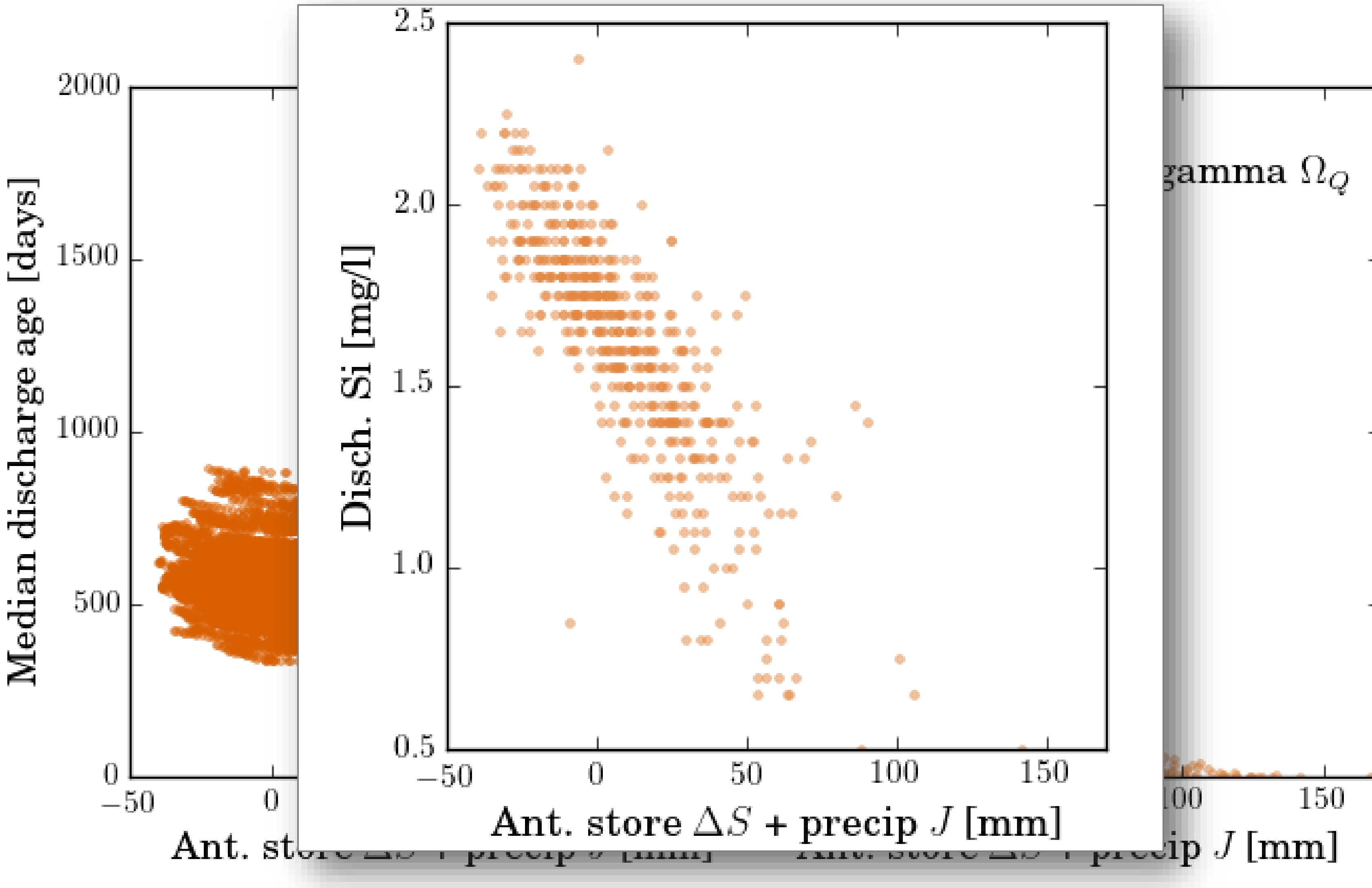
Fixed gamma (2 param)

Storage-dependent gamma (3 param)

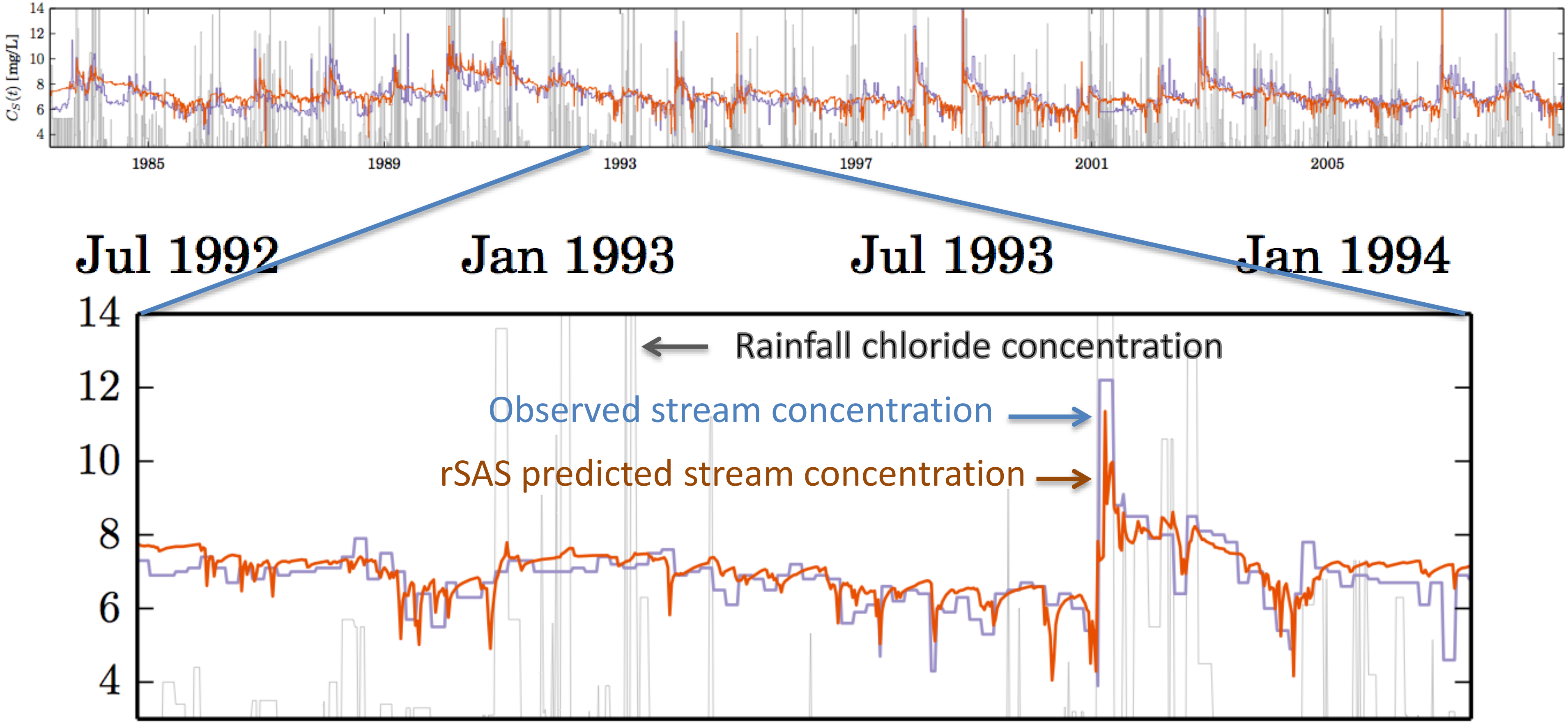
Nash-Sutcliffe Efficiency = **0.66** calibration **0.53** validation

Remember: 1 control volume, no hydrologic model

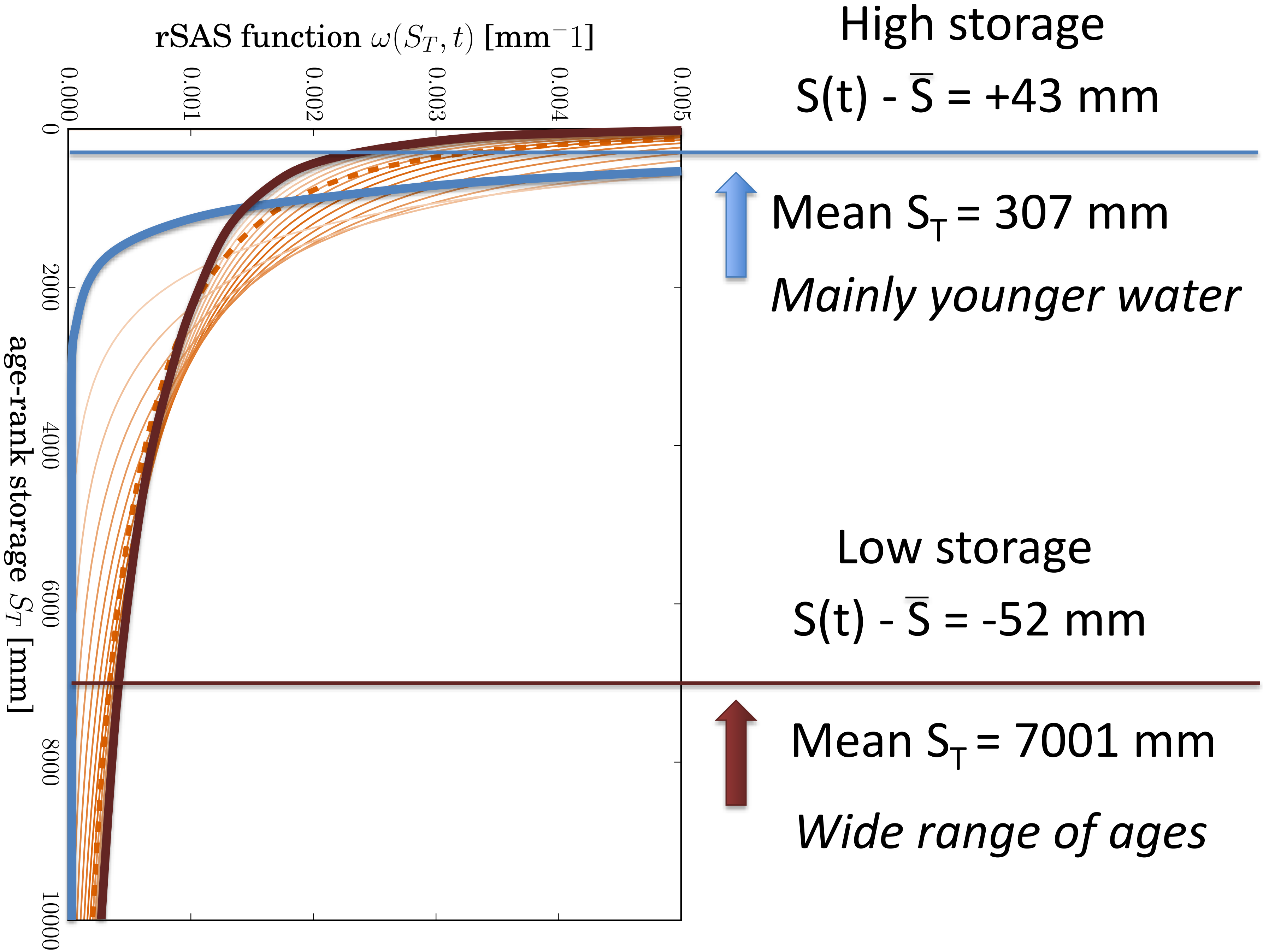
The difference between fixed and storage-dependent: age variability

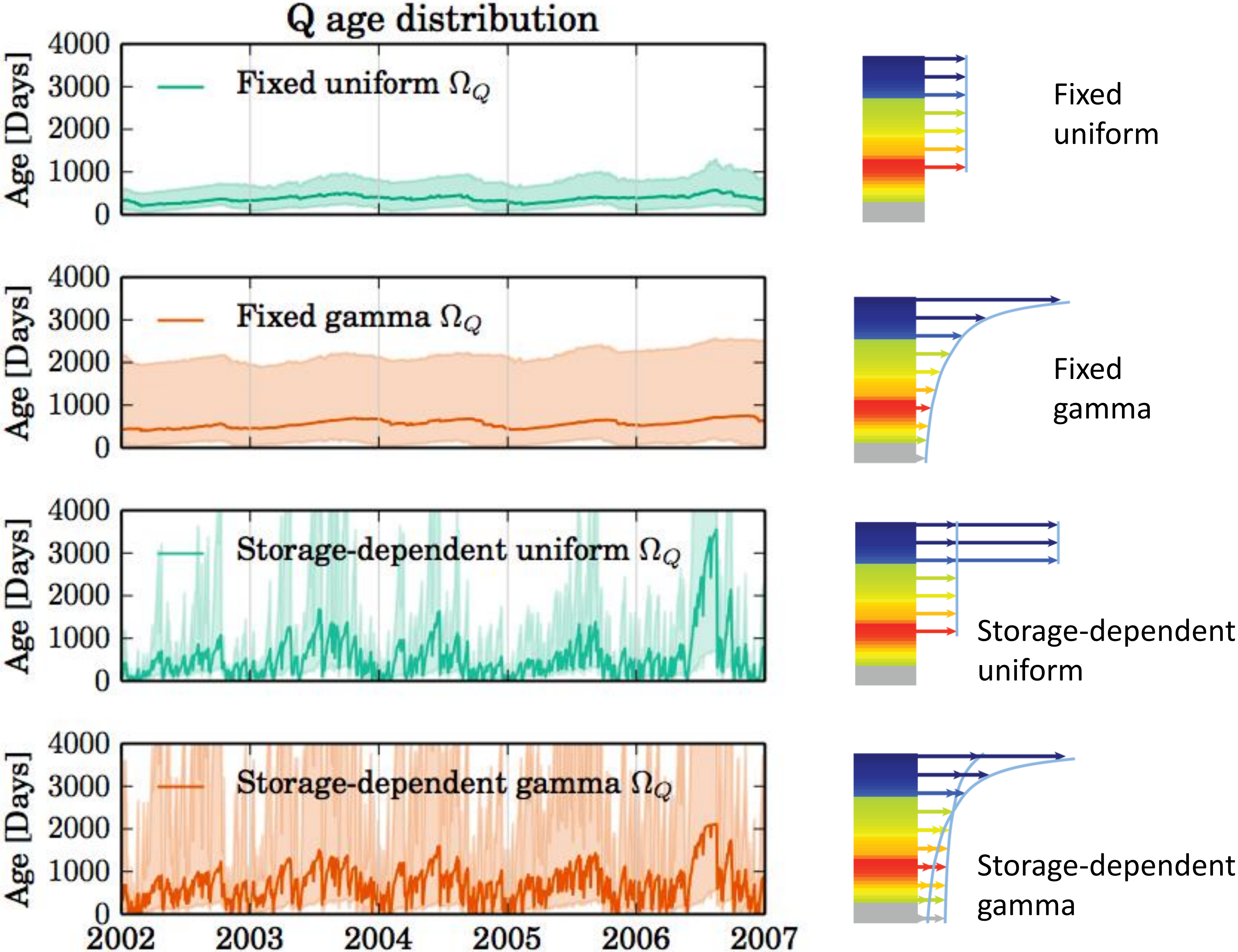


1 control volume, no hydrologic model

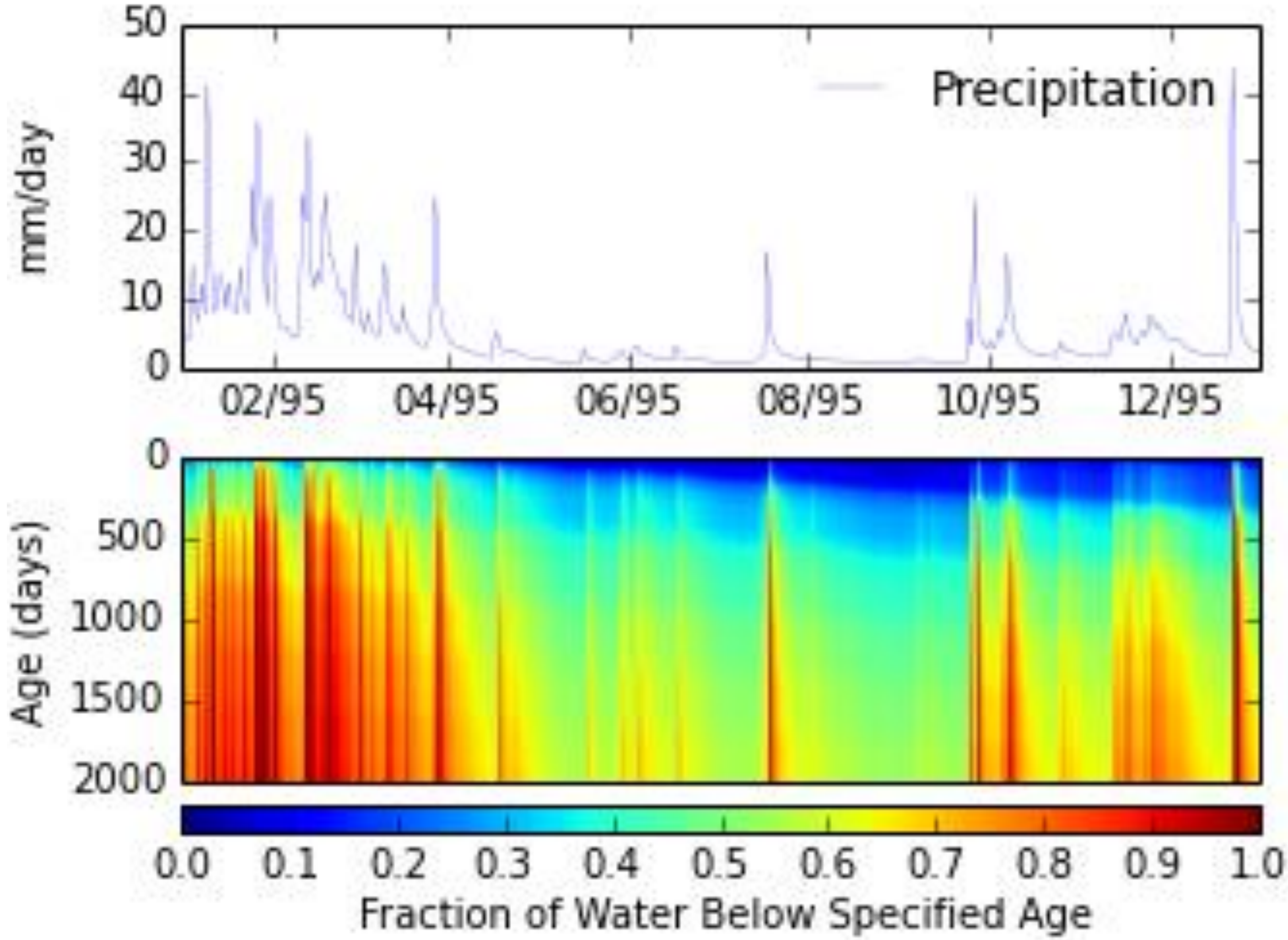


Calibration period (10 years) RMSE = 0.74 mg/L
Validation period (10 years) RMSE = 0.77 mg/L


rSAS function $\omega(S_T, t)$ [mm^{-1}]
0.000
0.001
0.002
0.003
0.004
0.005
age-rank storage S_T [mm]
0
2000
4000
6000
8000
10000
High storage
S(t) - $\overline{S}$ = +43 mm
Mean S_T = 307 mm
Mainly younger water
Low storage
S(t) - $\overline{S}$ = -52 mm
Mean S_T = 7001 mm
Wide range of ages

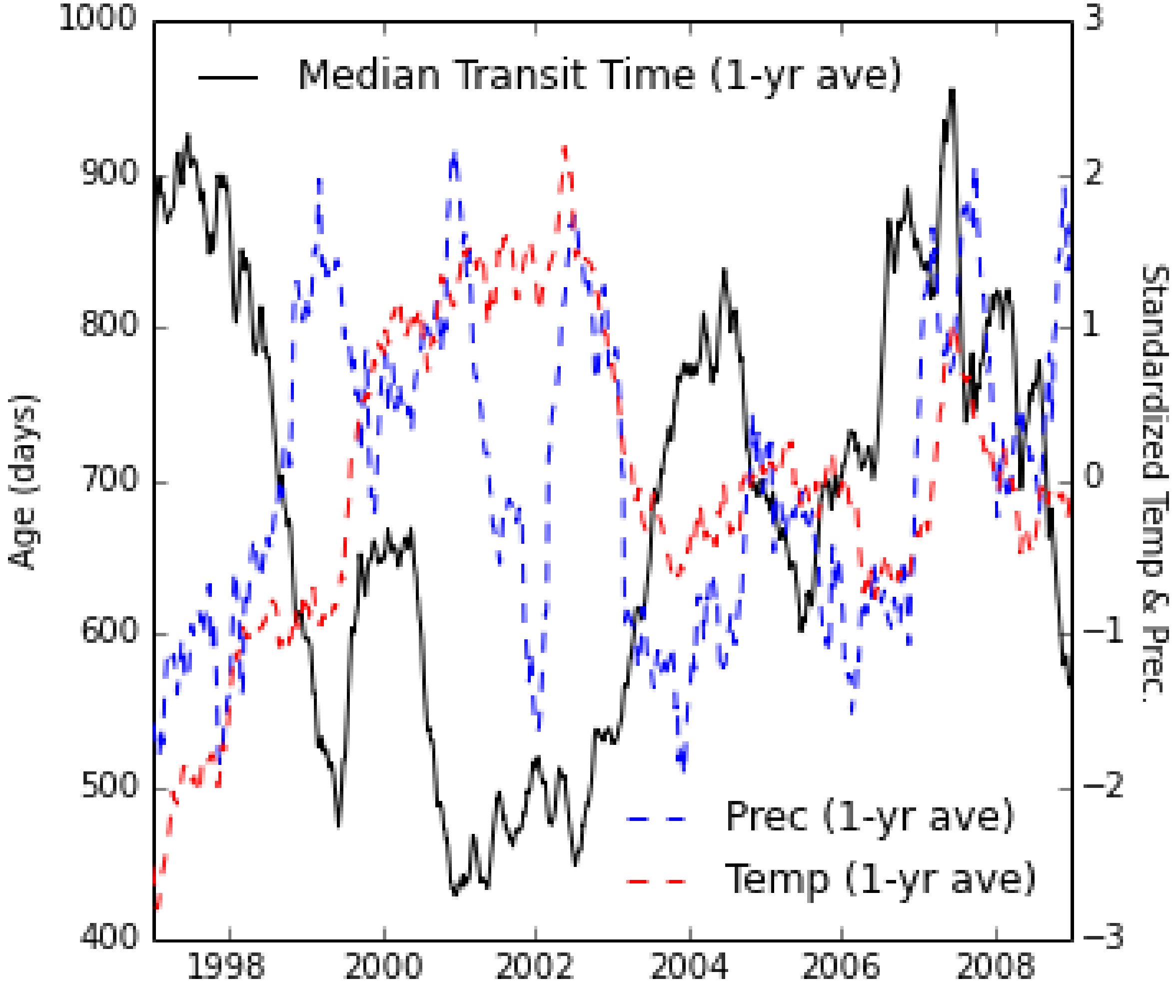

Q age distribution
Age [Days]
Fixed uniform Ω_Q
Fixed gamma Ω_Q
Storage-dependent uniform Ω_Q
Storage-dependent gamma Ω_Q
0
1000
2000
3000
4000
2002
2003
2004
2005
2006
2007
Fixed uniform
Fixed gamma
Storage-dependent uniform
Storage-dependent gamma

Event-scale climate sensitivity



Dano Wilusz

Inter-annual climate sensitivity



Dano Wilusz

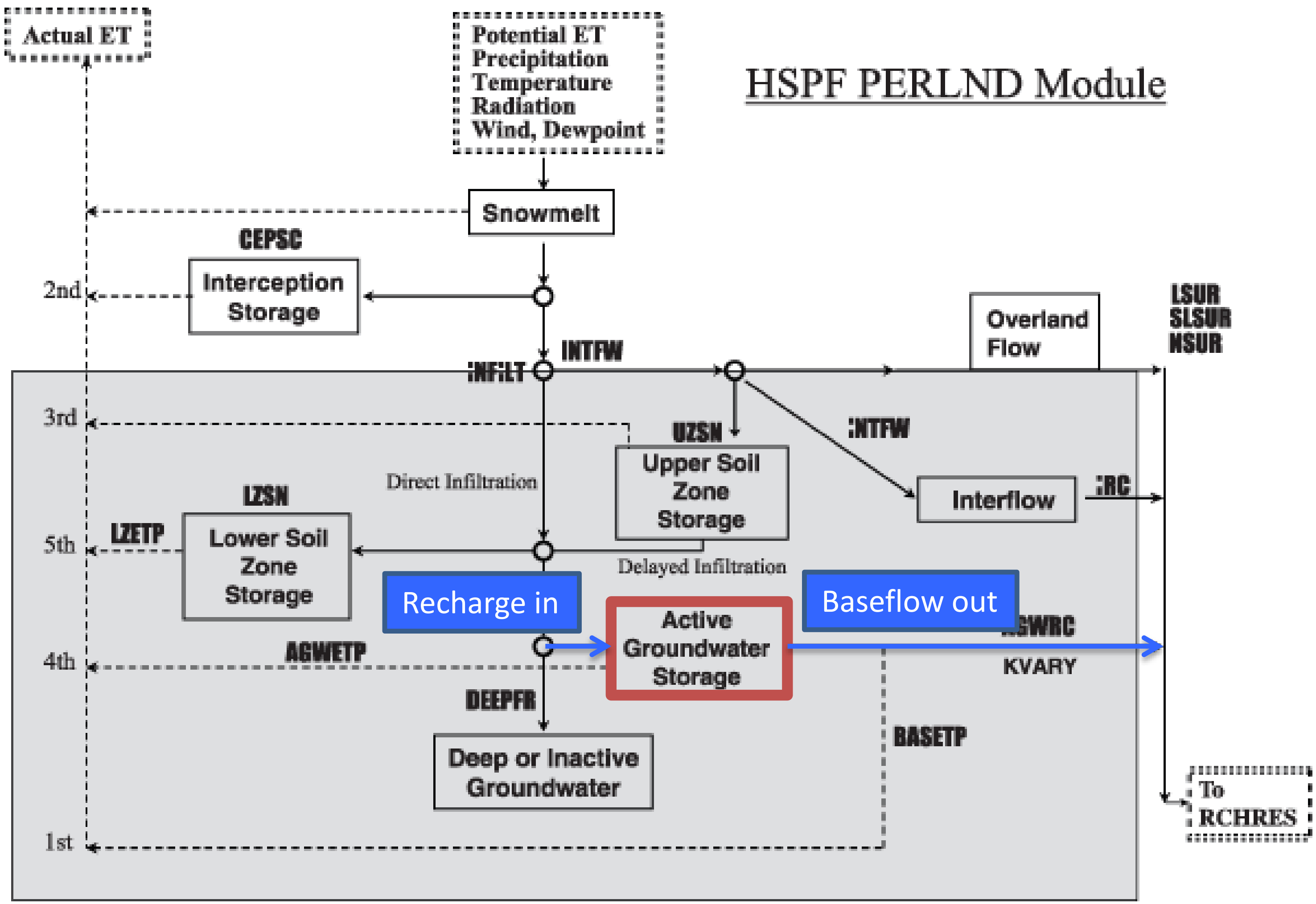
Water, Sustainability and Climate (Cat III)

Impacts of climate change on the phenology of linked agriculture-water systems

Johns Hopkins, UMCES, U Maine, Cornell, Virginia Tech

- Partnerships with EPA, USGS, USDA, Penn State
- Kickoff meeting this month at the CBPO, Annapolis

Possible rSAS implementation in CBP Watershed Model



“First-cut” parameterization linked to USGS groundwater modeling?

Recharge rate R

Modflow/Modpath

Land-water segment TTD for ages $T > 1$ year

Equivalent rSAS for $S_T > 1$ year of recharge

1-parameter calibration for shorter S_T

Regionalization at physiographic province level

Thanks!

- Co-PIs - Peter Troch (EAR), Bill Ball (WSC)
- Luke Pangle, Dano Wilusz, Qian Zhang, Shane Putnam, Ashley Ball, Minseok Kim, Holly Guest, Yifan Zhou
- National Science Foundation
  - EAR-1344664, EAR-1417175, CBET-1360415
- CUAHSI Pathfinder Fellowship

Harman, C. J. (2014), Time-variable transit time distributions and transport: theory and application to storage-dependent transport of chloride in a watershed, *Water Resour. Res.*, *in press*. DOI: 10.1002/2014WR015707